Dynamic Analysis of Pile Foundations: Effects of Material Nonlinearity of Soil

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ABSTRACT

Due to soil-structure interaction effects, the dynamic response of pile-supported structures is strongly dependent on the behavior of soil-pile system. A rational design of such structures requires adequate analysis of the soil-pile system. At strong ground excitations behavior of soil surrounding the piles is nonlinear. In this paper, a simplified and computationally efficient technique is presented to incorporate the material nonlinearity of soil in the dynamic analysis of piles. Approach is based on Green's function formulation and a hyperbolic model is used to define the nonlinear stress-strain relationship for the soil. Effects of material nonlinearity of soil, on the free field response, the seismic response and the impedance functions of pile foundation, are investigated. Analysis is performed both for a single pile and a pile group. It was observed that both seismic response and impedance functions are significantly affected by the nonlinear behavior of the soil media.

KEYWORDS: Soil-pile interaction, material nonlinearity, hyperbolic model, seismic response, impedance functions.

INTRODUCTION

The response of pile foundations is greatly affected by the behavior of soil media, in which piles are embedded. Considerable research has been conducted for the analysis of pile groups (e.g., Novak 1974, Wolf and Von Arx 1978, Kaynia and Kausel 1982, Gazetas 1984). In most of the literature, the behavior of the soil is assumed elastic. During strong ground excitation such as caused by earthquakes this assumption is not valid. The performance of highway bridges (founded on piles) in recent strong earthquakes such as Kobe (1995), Kocaeli (1999), Chi-chi (1999), Bhuj (2001) demonstrated that nonlinear behavior of soil media should be taken into account in the design of pile foundations.

A few researchers have incorporated material nonlinearity of soil in the dynamic analysis of pile foundations. Nogami et al. (1992) used a Nonlinear Beam on Winkler Foundation model in the time domain. In much of the previously published research, the nonlinearity in the soil is typically modeled using discrete system of mass, spring and dashpot. Using such models, it is difficult to properly represent damping and inertial effects of continuous semi-infinite soil media. Further full coupling in the axial and lateral directions is difficult to consider.

Recently few advanced plasticity based nonlinear soil models were employed in the time domain with finite element method to introduce material nonlinearity of soil. For example, Bentley and El Naggar (2000) considered Drucker-Prager soil model while Maheshwari et al. (2004 and 2005) employed HiSS soil model for this purpose. While these sophisticated models along with finite element technique are capable of modeling nonlinear behavior (plastic characteristics), however their computational demand is quite high. Further, these models are applicable for limited types of soils.

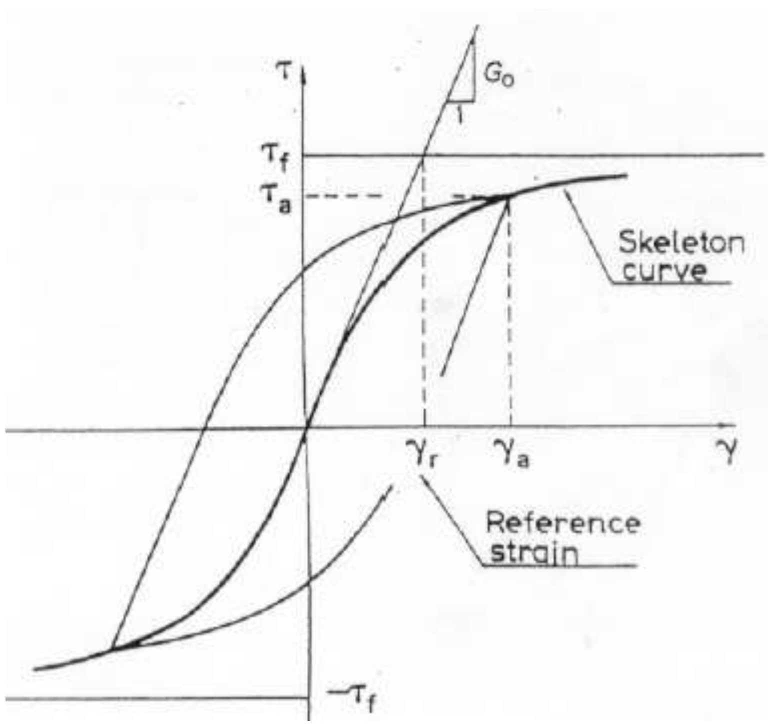


Figure 1. Hyperbolic Stress-Strain Relationship of Soil Media (Ishihara 1996)

During moderate earthquakes, shear strains in soil media fall in the medium range (10^{-5} to 10^{-3}) and the behavior of the soil becomes elasto-plastic. In this range of strains shear modulus and damping ratio, both of which depend on the level of strain are the key parameters to properly model the soil medium. In this range of strain behavior of soil medium can be best defined by hyperbolic stress-strain relationship (Fig. 1). Applicability of this model for representing different soils is widely demonstrated (e.g. Hardin and Drnevich 1972, Ishihara 1996). In this paper, this model is used to formulate the material nonlinearity of soil for the dynamic analysis of pile foundations.

For the linear pile analysis a three-dimensional rigorous approach based on boundary integral method (Green's function formulation) and proposed by Kaynia and Kausel (1982) is used. Hyperbolic soil model is incorporated using equivalent linearization technique (Watanabe 1978). Two types of sites namely homogeneous and inhomogeneous are considered in the analyses. First, effects of nonlinearity on free field response are observed. Then effects of nonlinearity on seismic response and impedance functions of soil-pile system are investigated by comparing linear and nonlinear (equivalent linear) responses. Both a single pile and a pile group are considered. It is assumed that there is no loss of bond between soil and pile.

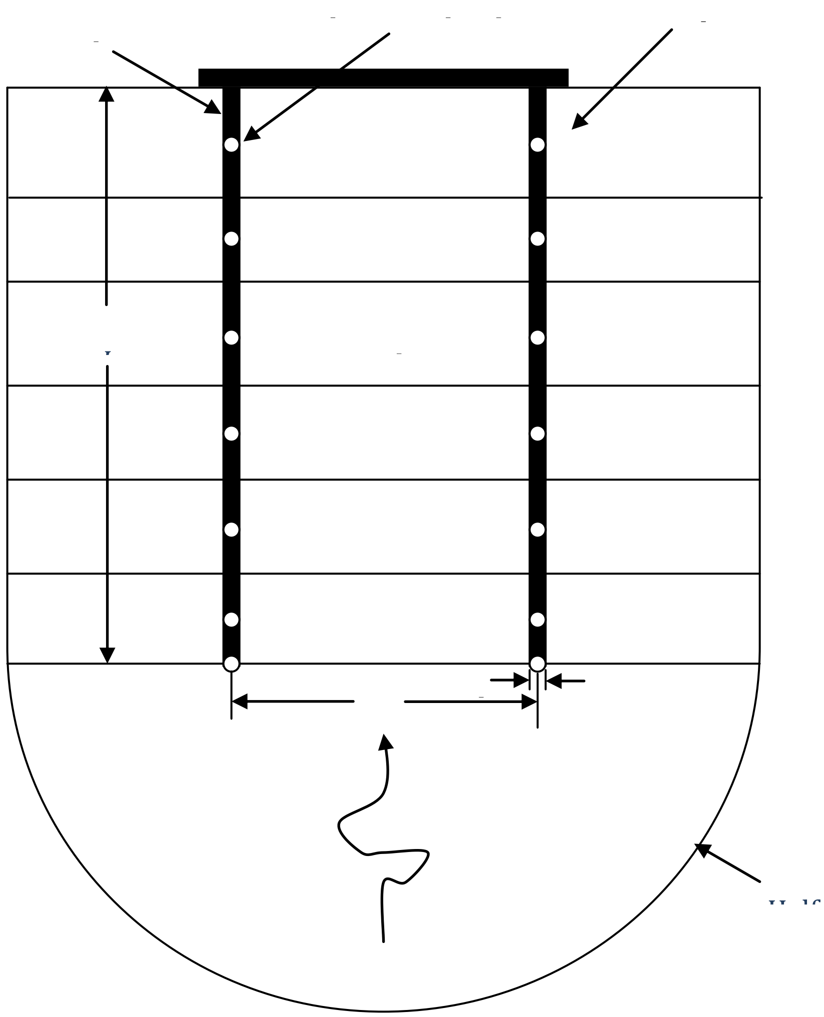


Figure 2. Proposed Model for the Soil-Pile System for Nonlinear Analysis

MODELING

Soil is a semi-infinite medium and modeled as a viscoelastic layered half-space (Fig. 2). Properties of soil may vary from layer to layer but they are assumed constant in a particular layer. Piles are also divided into segments with numbers and segment lengths matching that of the soil layers. The centroid of each segment and the pile tips define the number of nodes under consideration. Here for simplicity, it is assumed that pile ends (tips) are resting on the half-space and seismic excitation is due to vertically propagating shear waves.

Both a single pile and a 2*2 pile group are considered in the analyses. For the group, 4 piles in a square configuration is used with center to center pile spacing s equal to 5d, where d is the diameter of the piles. Also for the group it is assumed that all pile heads are connected through a rigid massless pile cap.

FORMULATION

Three-dimensional dynamic analyses for the soil-pile system are performed for two situations. In first case, behavior of soil is assumed elastic. This linear analysis also used to determine the maximum shear strains in different layers of soil. In second case, the properties of the soil (in different layers) are modified using hyperbolic stress-strain relation. Second case is an iterative process (equivalent linearization) and repeated till soil properties get converged. These are described in detail below.

Linear Analysis

A rigorous three-dimensional approach proposed by Kaynia and Kausel (1982) has been used for the linear analysis. This approach fully takes into account the coupling between the horizontal and vertical modes of vibration. Effects of pile-soil-pile interaction, which is very important in the case of pile group, are also considered. A brief formulation for this approach is presented here, details can be found in Kaynia (1982).

Three basic wave equations are solved through Fourier and Hankel transformations, and Green's functions. Thus the displacement fields due to uniform barrel and disk loads associated with soil-pile interaction forces are computed. These functions yield the dynamic soil flexibility matrix, which is combined with the pile flexibility matrix derived by solving the beam equations. After certain manipulations and applying boundary conditions, a relationship relating forces at pile heads (and pile tips) with the displacements at these points are obtained which is described as:

$$\mathrm{P_e} = \mathrm{K_e U_e} + \overline{\mathrm{P}}_\mathrm{e} \tag{1a}$$

where P_e and U_e are the force and displacement vector respectively, referred to the end of the piles (i.e. pile heads and pile tips). K_e is an equivalent stiffness matrix representing the dynamic stiffness of the soil-pile system and $\overline{P}_e$ is a load vector due to seismic effects; these are defined as:

$$\mathrm{K_e} = \left[\mathrm{K_p} + \psi^\mathrm{T}(\mathrm{F_s} + \mathrm{F_p})^{-1}\psi\right] \tag{1b}$$

$$\overline{P}_e = -\psi^T (F_s + F_p)^{-1} \overline{U} \tag{1c}$$

Where $\overline{U}$ represents the seismic displacement in the medium which is equal to free field displacement, and matrices: F_s = Soil flexibility matrix when there is no pile in the medium; F_p = Pile flexibility matrix for clamped end piles; K_p = Pile stiffness matrix relating forces at the end of piles with end displacements; ψ = Shape function matrix relating displacement at any point on pile with the end displacements.

Shear Strains

The shear strains in the soil medium are required to check the nonlinear behavior of soil. Initial values of shear strains are determined assuming the elastic behavior of soil. Determination of the strains at a desired point requires the displacements at the point under consideration, as well as in the near vicinity of this point. This is performed using equations 1 with following additional steps:

Soil-Pile-Interaction Force

First using equation 1 the degrees of freedom related to pile tips are eliminated by applying appropriate boundary conditions at pile tips (e.g. zero displacements for end bearing piles; zero moments for floating piles). Next using the compatibility conditions between the pile heads and pile cap the seismic response of the pile foundation is determined using equation 1a with the fact that the resultant of pile head forces on pile foundation is zero. Subsequently, using compatibility conditions between the pile heads and pile cap, the displacements at the end of the piles U_e are determined. Then force vector P at soil-pile interface can be determined from:

$$P = (F_s + F_p)^{-1} (\psi U_e - \overline{U}) \tag{2}$$

Displacements in the vicinity of Pile

Once the force vector P is known, the displacement at any point in the vicinity of pile in all the three directions can be computed using the relation:

$$U = \overline{U} + F_s' P$$

where F_s' denotes the soil-flexibility matrix derived for the distance at which displacements are desired when the source point of disturbance is the axis of the pile. Using Eq. 3, the displacements in soil media for a homogeneous site are derived as shown in Fig. 3 (shown for top five layers, out of 15 layers considered) where the horizontal distance is normalized with the radius r_0 of the pile. It can be seen that displacement decreases away from the pile.

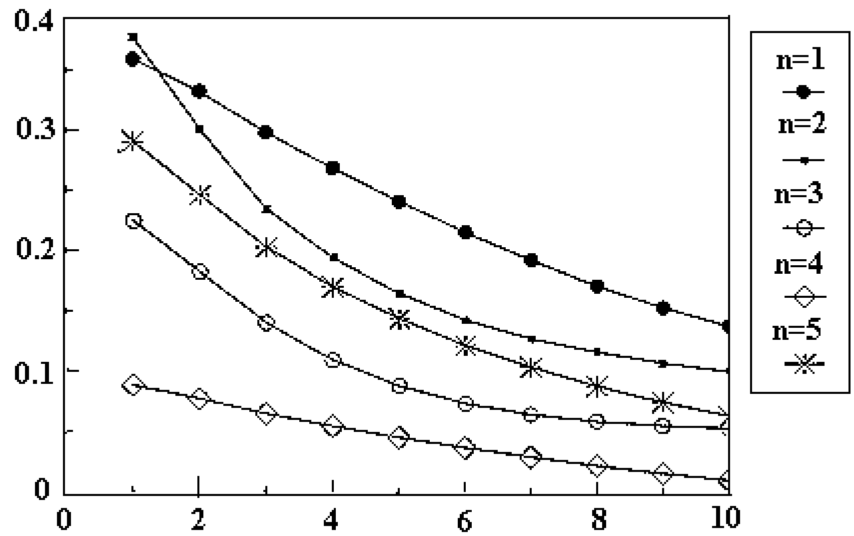


Figure 3. Variation of Lateral Displacement in Different Layers with Horizontal Distance

Shear Strains in the Soil Media

Once the displacements are known at the various points in different directions, the shear strains for the planes perpendicular to *z* axis (vertical) are computed by the following relations:

$$\gamma_{xz} = \frac{u_2 - u_0}{\Delta z} + \frac{w_1 - w_0}{\Delta x} \tag{4a}$$

$$\gamma_{yz} = \frac{v_2 - v_0}{\Delta z} + \frac{w_3 - w_0}{\Delta y} \tag{4b}$$

where *u*, *v* and *w* are the displacements in the *x*, *y* and *z* directions, respectively, and the subscript denotes the location of points with subscript ($_0$) for the point where strains are calculated. For computation, $\Delta x = \Delta y = \Delta z = r_0$ is taken, where r_0 is equal to the radius of the pile. Out of these shear strains, the larger one is selected to carry out the iterations. For horizontal vibration (caused by vertically propagating shear waves) the motion in the horizontal (*x*) direction being considerably higher, the major shear component comes out to be the first one.

Fig. 4 shows the variation of shear strains (major) in the top five layers with the horizontal distance at a particular frequency. It can be observed that the shear strain diminishes as one goes away from the pile, hence the strain calculated at a distance equal to 1.5 r_0 is taken as a representative strains in the soil media where strains are maximum in most of the layers. Strains just near the pile surface may be higher (than those computed at $r = 1.5\ r_0$) but it may have influence of the surface of the pile and for iterations strains in soil are required (not in pile), therefore this distance ($r = 1.5\ r_0$) is justified.

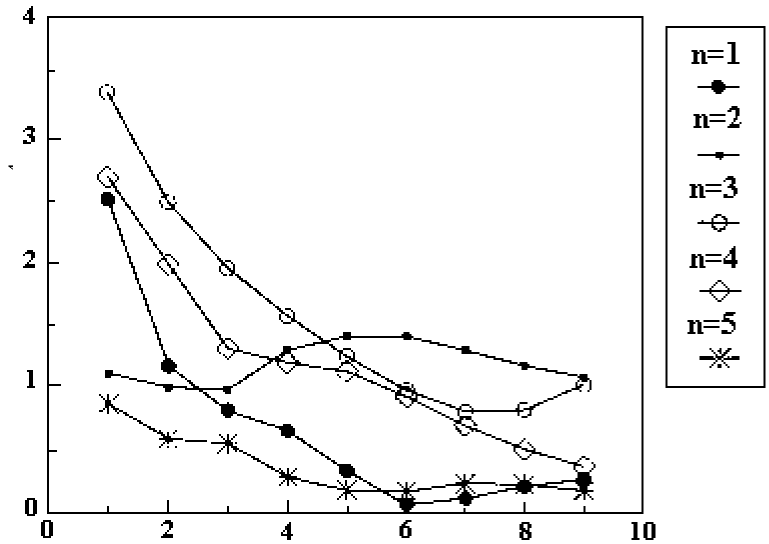


Figure 4. Variation of Shear Strain in Different Layers with Horizontal Distance

Nonlinear Soil Model

Nonlinearity of soil is treated using hyperbolic model, defined by following equations (Hardin and Drnevich 1972):

$$\frac{G}{G_{\max}} = \frac{1}{1+\gamma/\gamma_r} \tag{5a}$$

$$\frac{D}{D_{\max}} = \frac{\gamma/\gamma_r}{1+\gamma/\gamma_r} \tag{5b}$$

Where G and D represent the shear modulus and damping at a particular strain γ, while G_{max} and D_{max} represent the maximum values of G and D, respectively. γ_r represents the reference strain for the given soil media (Fig. 1), Ishihara (1996).

Equivalent Linearization Technique

To perform the seismic analysis for harmonic excitations, seismic displacements in soil are computed at the midpoints of each layers with the given soil properties. Next, shear strains are computed in soil media in each layer at each frequency of loading and the process is repeated for all the frequencies under consideration. Out of these values of shear strains, maximum one (for each layer) is selected for the iterations. Using equations 5, new properties (shear modulus and damping) of the soil media for each layer are computed for a strain equal to 2/3 of the maximum shear strain (Kramer 1996). Since the properties of the soil medium are changed now, shear strains are calculated again for the input motion. Properties are again modified for new strain values. The process is repeated until the properties of the soil medium get converged. The nonlinear (equivalent linear) response is the response computed using the converged properties of the soil medium. The criteria of convergence of soil properties are determined from the difference in the shear modulus in two consecutive cycles, and assumed to be stable if:

$$\left|\frac{G^i - G^{i-1}}{G^i}\right| \leq 0.05 \qquad (6)$$

Computerization

A FORTRAN code was developed to perform the analysis. The program has different modules, e.g. for free field response, impedance functions, pile head response etc. As the formulation for the analysis is in the frequency domain, to perform the analysis for transient motion (a real earthquake), the FFT and inverse FFT algorithm have been employed. Computation is done in double precision, and dynamic stiffness of soil that is generally complex requires use of the complex data type. Its real part represents the spring stiffness and imaginary part represents damping. Complex dynamic stiffness of the soil requires most of the other quantities such as impedance function and response to be complex.

VERIFICATION OF THE MODEL AND ALGORITHM

As a rigorous approach is used, verification of the model and algorithm developed is imperative. This is performed by comparing the results obtained from present algorithm (for linear analysis) with those published in literature. Fig. 5 shows the variation in dynamic stiffness with frequency for horizontal and vertical mode of vibration for a single pile and a 2*2 pile groups (with three different spacing). Both real and imaginary parts are shown in the dimensionless form. When these results are compared with those presented by Kaynia and Kausel (1982) a good agreement is found. Trend of the results are in very good agreement. Verification of model and algorithm was also performed for rocking and torsional modes of vibrations as well as for seismic response and all found in good agreement (Maheshwari 1997).

DATA USED IN COMPUTATION

Two types of sites namely homogeneous and inhomogeneous are considered in the analysis (Fig. 6). For the homogeneous site, the initial shear modulus and the damping ratio from top to bottom is assumed constant. For inhomogeneous site, the initial shear modulus is assumed linearly increasing from top to bottom. The properties of the half-space are assumed to be the same for both sites.

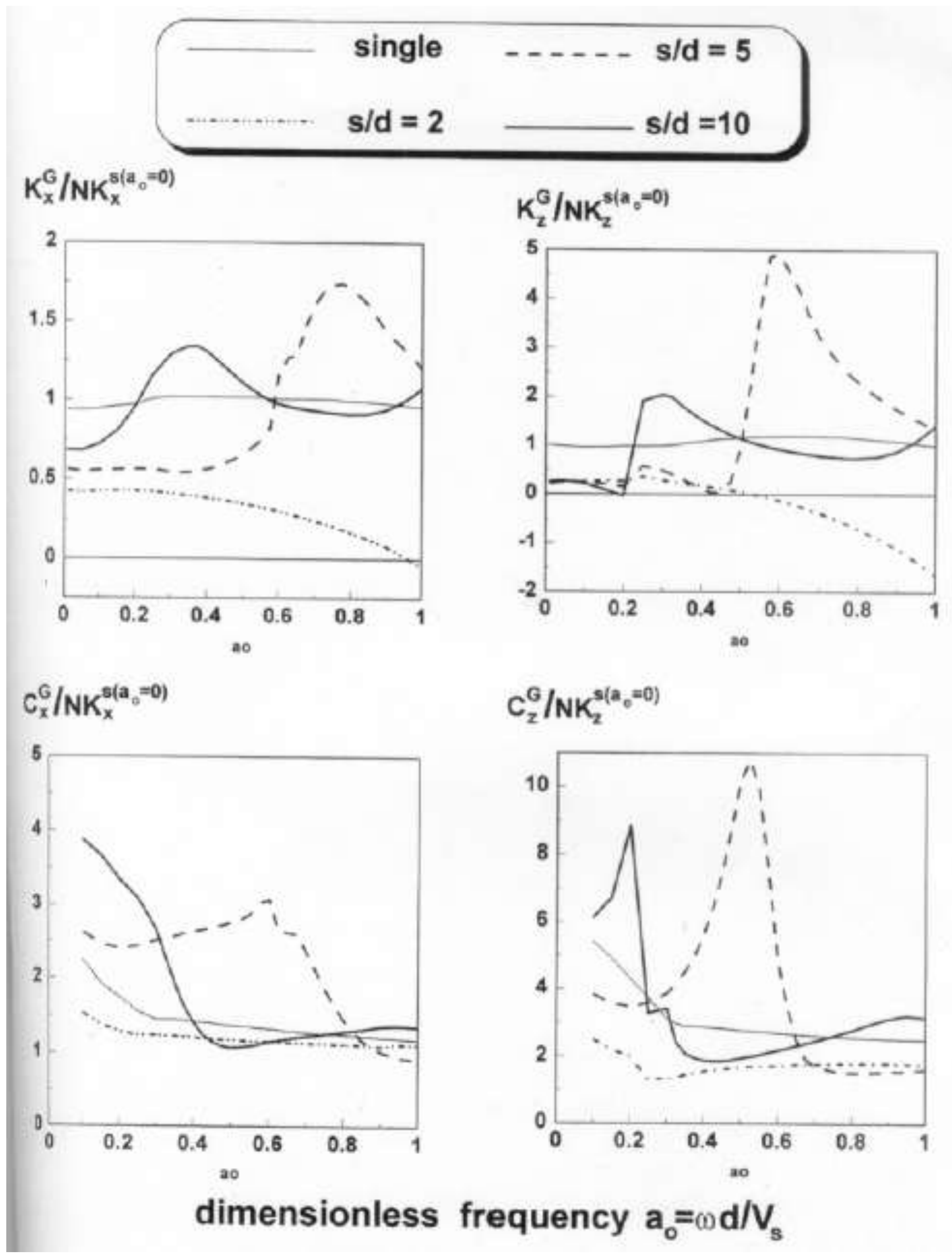


Figure 5. Variation in Horizontal (subscript x) and Vertical (subscript z) Dynamic Stiffnesses of a Single Pile and 2*2 Pile Groups (for three different spacings) with Frequency of Excitation (a_0). Top figures show real part (stiffness -K) and bottom figures show imaginary part (damping - C).

The soil is assumed to be typical sand with the following properties:

$$E_s = 50\,\text{MPa}; \quad D = 0.05; \quad \rho_s = 1680\ \text{kg/m}^3; \nu = 0.4;$$

where E_s, D, ρ_s and ν are the initial Young's modulus (at ground surface), initial damping ratio, the mass density and Poisson's ratio for soil, respectively. For the hyperbolic model, following constants are used:

$$G_{max} = 60\,\text{MPa}; \quad D_{max} = 0.29; \quad \gamma_r = 3.6\times10^{-4};$$

The piles are assumed to be of concrete with the following properties:

$$E_p = 25\,\text{GPa}; \; \rho_p = 2400\ \text{kg/m}^3; L = 15\,\text{m}; d = 1\,\text{m};$$

where E_p, ρ_p, L and d are the Young's modulus, the mass density, length and diameter of pile, respectively.

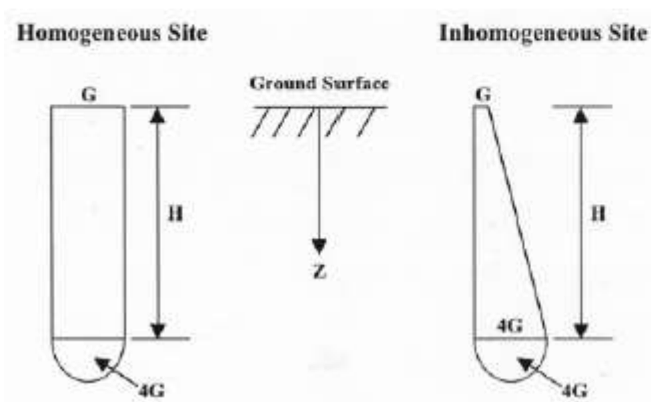


Figure 6. Soil Profiles used in the Analyses.

EFFECTS OF NONLINEARITY

Linear and nonlinear (equivalent linear) responses of the pile foundations are computed. Results are presented in following three sub-sections:

Free Field Response

Since nonlinearity of the soil media is the major concern, and the behavior of the free field soil greatly affects the overall response of the soil-pile system, therefore free field response is first investigated. This response is not only frequency dependent but also depends very much on the properties of the site.

Figure 7 shows the linear and nonlinear free-field response (at the ground surface) when a harmonic excitation (of amplitude 0.2 cm) is applied laterally at the base of an inhomogeneous site. It can be seen that with reference to the natural frequency of the soil stratum, at low frequencies nonlinearity increases the response as much as 20% while at high frequencies it decreases the same as much as by 50%. Also, the peak value of nonlinear response occurs relatively at low frequency due to softening of soil. There is a cutoff frequency at which both linear and nonlinear responses are equal.

To verify these results, the effect of shear modulus and damping is investigated in Figs. 7b and 7c, respectively. In Fig. 7b, the response is derived for the converged value of the shear modulus while setting damping to original value. In Fig. 7c, the response is derived for the modified value of the damping while setting shear modulus to original value. It is clear from these figures that the shear modulus (which decreases during linearization process) increases the response but material damping (which increases during linearization process) decreases the response. Therefore, it is the combined effect of these two parameters that governs the response in Fig. 7a.

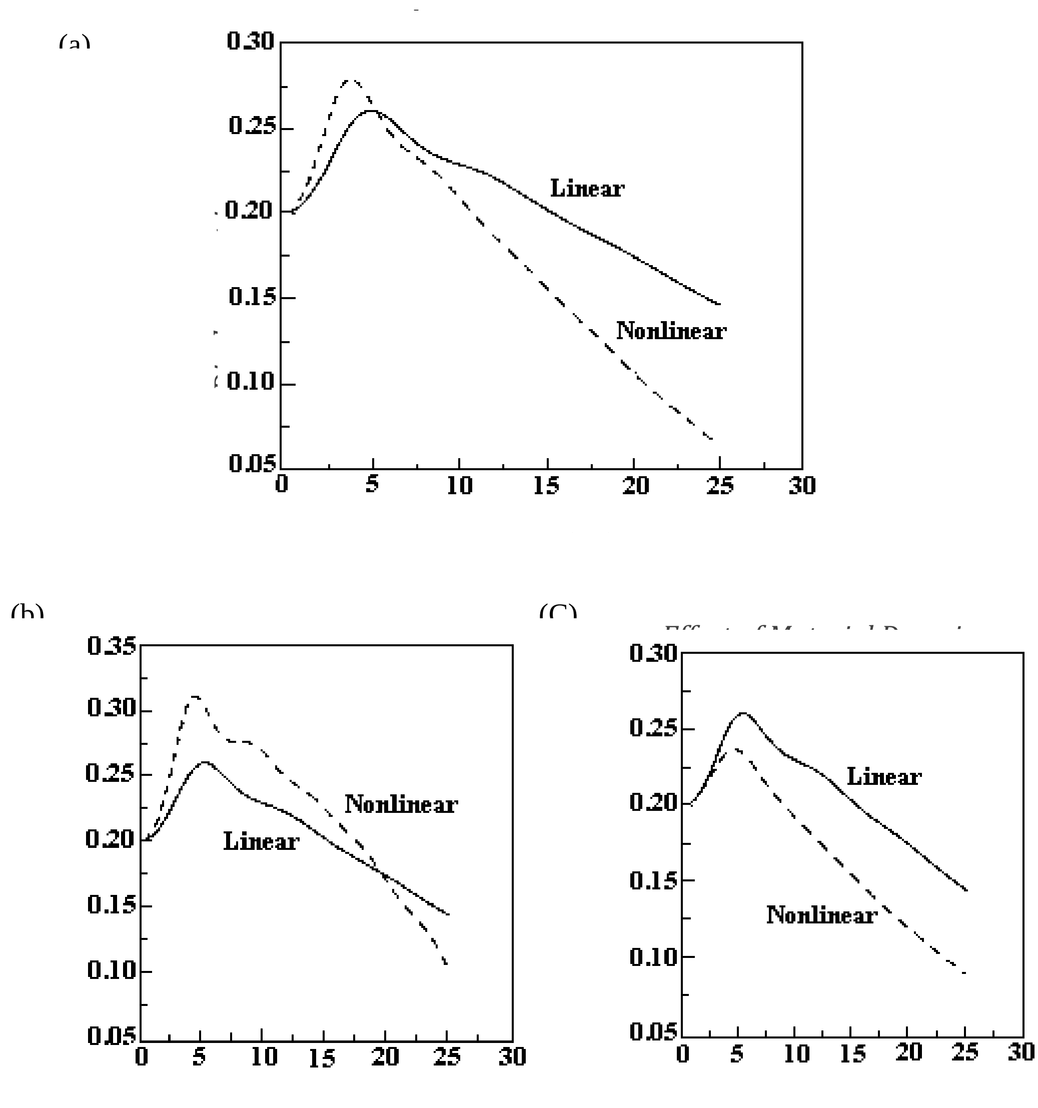
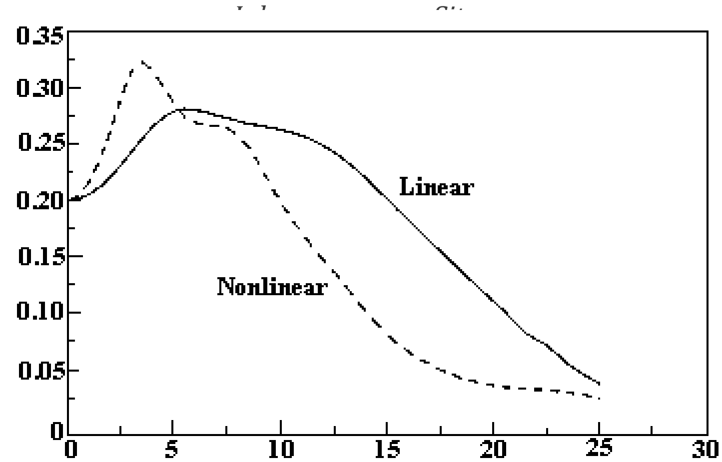


Figure 7. Linear and Nonlinear Free Field Responses of Soil with Frequency of Excitation: (a) Response for an Inhomogeneous Site (b) Effect of Shear Modulus (c) Effect of Damping

Seismic Response of the Pile Foundation

The seismic excitation causes horizontal displacement and rotation of the pile cap. The amplitude of the applied harmonic excitation should be high enough to cause shear strains such that it induces the material nonlinearity in soil. Figs. 8a and 8b shows the linear and nonlinear response (at pile head) of a single pile for inhomogeneous and homogeneous sites, respectively. The trend of the results is similar to that observed with the free field response. Nonlinearity is increasing the response at lower frequencies and decreasing at higher frequencies. Also, due to nonlinearity, the peak value of the response occurs at a lower frequency. It can also be observed that effect of nonlinearity is higher for inhomogeneous site where response is increased by about 30% at low frequencies of excitation. For homogeneous site many peaks are introduced in the response due to sharp change in the stiffness at the junction of soil layers and half space (Wolf 1985). These peaks appeared to decrease the effect of nonlinearity.



(b)

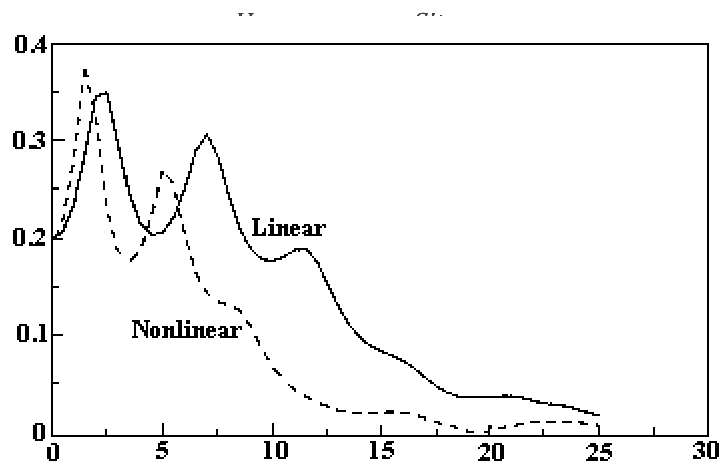
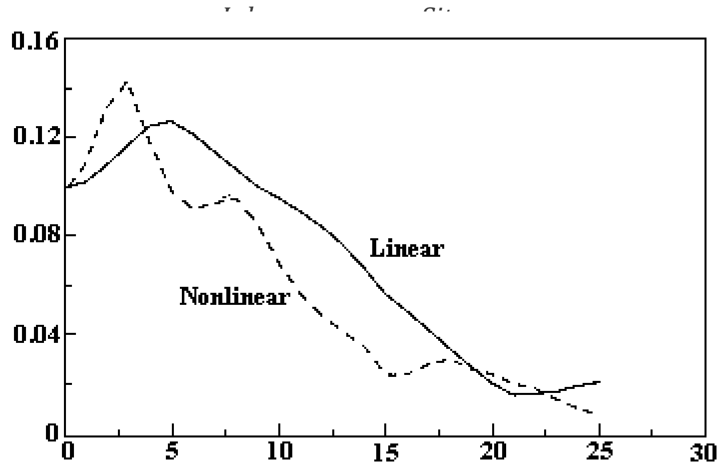


Figure 8. Linear and Nonlinear Seismic Response for a Single Pile (a) For an Inhomogeneous Site (b) For a Homogeneous Site

Figs. 9a and 9b show this comparison for a 2*2 pile group where the response is observed at the pile cap. Spacing between piles is assumed to be five times the diameter of the pile. The results have similar trend, as was the case for a single pile, except that more peaks are introduced due to interaction effects. From Figs. 8a,b and 9a,b it can also be observed that effect of material nonlinearity is larger for a single pile than for a pile group, particularly for a homogeneous site. It appears that interaction effect among piles is diminishing the effect of nonlinearity. Similar conclusion was derived by Maheshwari et al. (2004) using a finite element model in the time domain.

(a)



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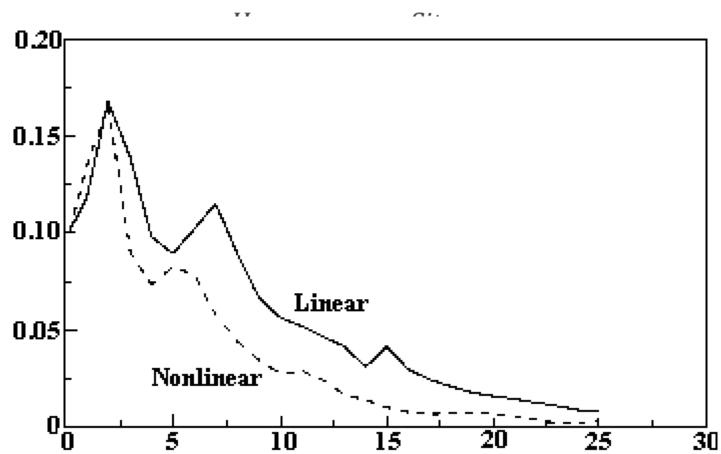


Figure 9. Linear and Nonlinear Seismic Response for a 2*2 Pile Group (a) For an Inhomogeneous Site (b) For a Homogeneous Site

Impedance Functions

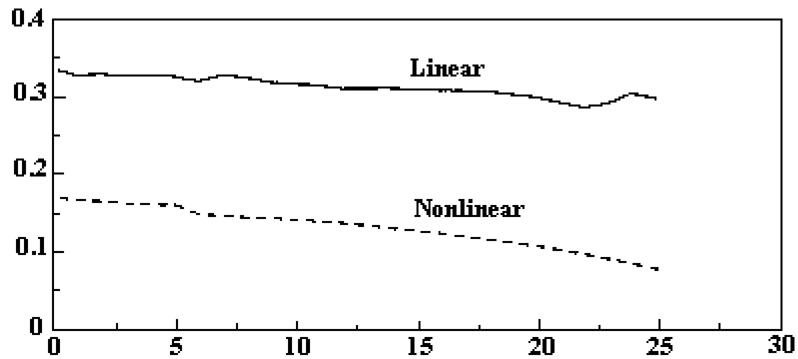
Impedance functions are the dynamic stiffness of the soil-pile system at the pile cap. The derivation of these functions does not require seismic motion, and thus free field motion is not needed. A harmonic excitation of given force amplitude is applied at the pile head in the direction in which impedance function is sought. The amplitude of this excitation should be high enough to cause nonlinearity in the soil. Impedance functions can be obtained by applying a load (in a specified direction) on the pile head or pile cap and noting the complex displacement (in the direction of the load) at the same point. The complex impedance function is defined as:

$$S(\omega) = P_0 / U_0 \tag{7a}$$

where P_0 and U_0 are the amplitude of the force excitation and complex displacement amplitude, respectively for a particular direction for which impedance function is sought. The impedance function obtained from Eq. 7a is a complex quantity and can be separated in to real (corresponding to stiffness) and imaginary parts (corresponding to damping). Both are frequency dependent i.e.

$$S(\omega) = k(\omega) + ic(\omega) \tag{7b}$$

(a)



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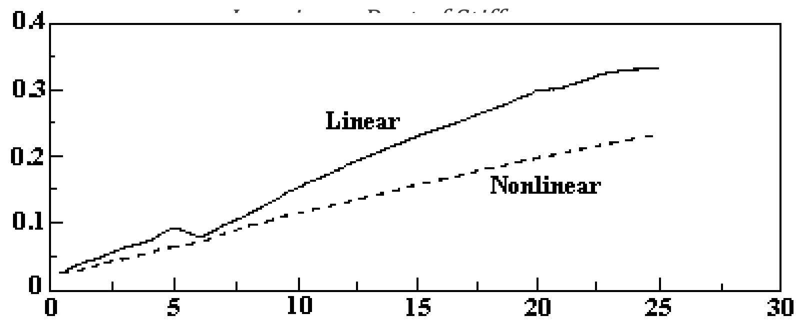

(c)

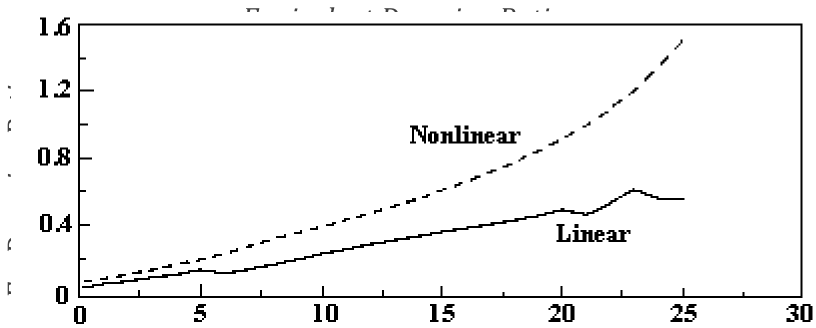

Figure 10. Linear and Nonlinear Impedance Functions of a Single Pile (Inhomogeneous Site)

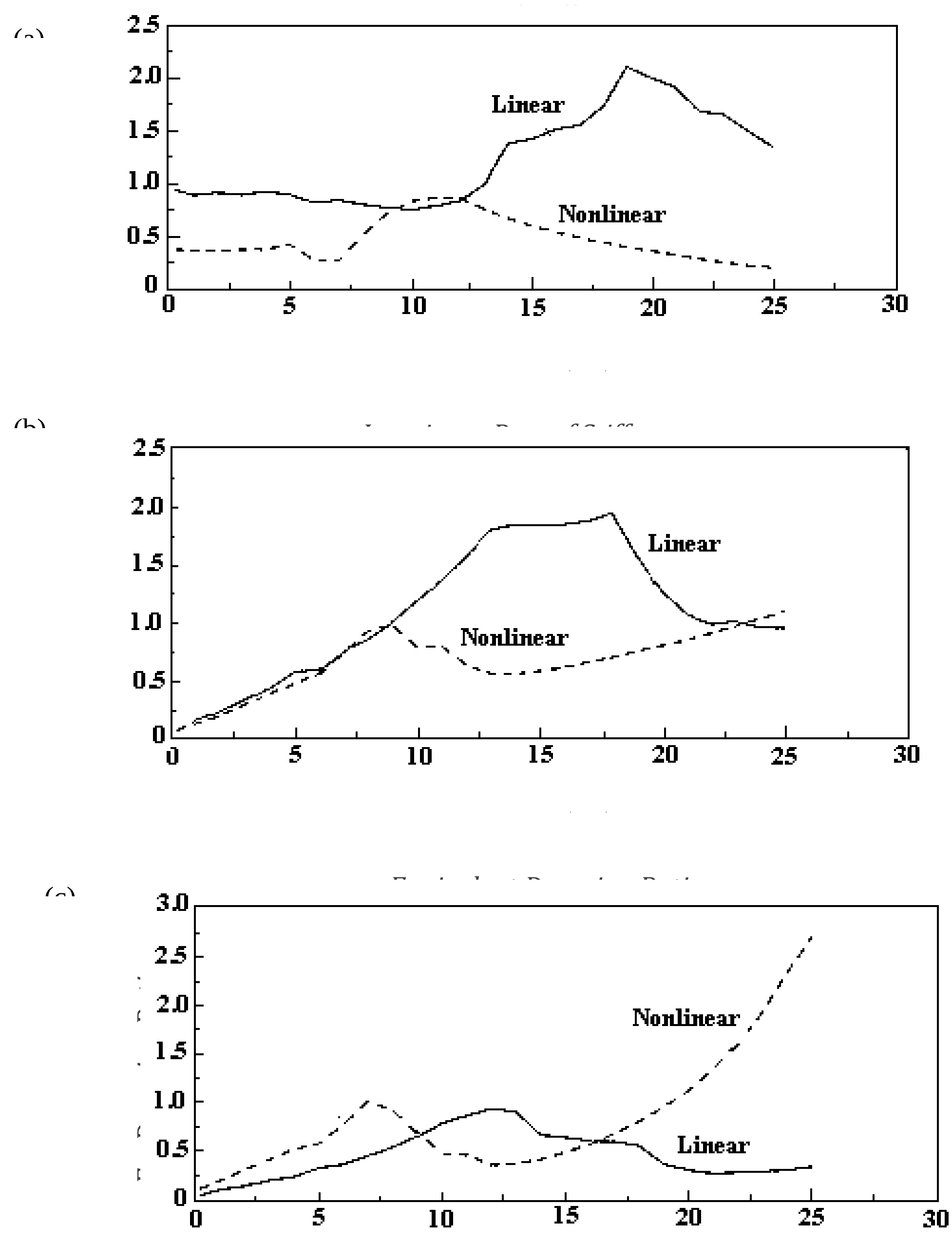

Figure 11. Linear and Nonlinear Impedance Functions of a 2*2 Pile Group (Inhomogeneous Site)

Alternatively this impedance function can be written as:

$$S(\omega) = k(\omega)(1 + 2i\beta') \tag{7c}$$

where β' is equivalent damping ratio, and using Eqs. 7b and 7c, it can be derived as:

$$\beta' = c(\omega)/2k(\omega) \tag{7d}$$

This equivalent damping ratio is frequency dependent and represents a measure of the equivalent damping (it includes effect of both material and radiation damping) in the soil-pile system, in contrast to the frequency independent hysteretic damping ratio of the soil media only.

Fig. 10 shows the real and imaginary parts of the impedance functions for lateral vibration of a single pile for both the linear and nonlinear cases. K_{sx} represents the dynamic stiffness of the soil-pile system of a single pile referred to its head. It can be noted from this figure that nonlinearity decreases both real and imaginary parts of the soil-pile system. But reduction in the real part (stiffness) is much larger compared to that in the imaginary part (damping), where nonlinearity is reducing the stiffness as much as by 60%. Also it can be observed that stiffness is reducing at all frequencies of excitation and variation with frequency is not significant as observed for the seismic response. Similar observation was made by Maheshwari et al. (2004). Also Fig. 10c shows the equivalent damping ratio (derived using equation 7d) for the linear and nonlinear cases. As expected from the real and imaginary part, this has higher values for the nonlinear case as compared to the linear case at all frequencies.

Figure 11 shows the impedance functions for a 2*2 pile group with pile spacing five times the diameter of the pile. Results show the same trend as observed in the case of a single pile but at low frequencies effect of nonlinearity was not significant. Also the equivalent damping ratio is higher for nonlinear case except for the certain frequencies where there is not much change in the real part of the stiffness.

These results can be explained by the fact that the softening of soil occurs due to nonlinear effects. When nonlinear effects of the soil are considered, then the value of shear modulus decreases for higher values of shear strain decreasing the impedance functions of the soil-pile system. At higher values of strains, the damping ratio of the soil media increases, increasing equivalent damping ratio of the soil-pile system.

SUMMARY AND CONCLUSIONS

In this paper a new approach is proposed to model the material nonlinearity of soil media for dynamic analysis of pile foundations. The following conclusions may be derived with the results presented:

Effects of nonlinearity on the free field response depend very much on the frequency of excitation. At low frequencies response is increased as much as by 20%. This free field response has a significant effect on the total response of the soil-pile system.

In general at lower frequencies (as compared to the natural frequency of the soil-pile system), material nonlinearity of the soil increases the seismic response as much as by 30%. At higher frequencies the reverse is true. Also, the peak in the nonlinear response occurs at a lower frequency as compared to the linear case. This is valid for a single pile as well as for a pile group.

1. Material nonlinearity reduces both the real and imaginary parts of the dynamic stiffness but the reduction in the real part is comparatively larger. Due to nonlinearity, the equivalent damping ratio

of the soil-pile system increases. Effect of nonlinearity on the real part of the dynamic stiffness is not much changed with frequency of excitation.

2. In general, effect of nonlinearity was greater for a single pile, compared to a 2*2 pile group. It appears that group effect diminishes the effect of nonlinearity.

Since seismic response and dynamic stiffness are very important parameters in the design of pile foundations, analyses presented here may have wide practical significance. For example, the low range of frequency where seismic response is increased as much as 30% due to nonlinearity is the range of interest for earthquake loading for a soil-pile system. The results presented have direct implications in the design of pile foundations or in general design of pile supported structures. The design will be more rational and safe if the effects of material nonlinearity are considered.

ACKNOWLEDGEMENT

The research work presented here was performed at Saitama University, Japan and supported by Monbusho fellowship for doctoral research by the Ministry of Education, Science, and Culture, Government of Japan. This support is gratefully acknowledged.

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